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ON A SIMPLE RELATION OF EXACT AIRFOIL THEORY

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Exact theory for incompressible potential flow yields the following relation for the distribution of surface velocities about a two-dimensional airfoil:[1,2]

$$U = k[\sin(\varphi+\alpha) + \sin(\alpha+\beta)] \tag{1}$$

where U is the velocity made dimensionless with respect to free stream, k is a function of airfoil geometry only, φ the angular coordinate on a circle conformally related to the airfoil, α the angle of attack, and β the negative of the angle of zero lift. By simple trigonometry, an interesting relation can be derived which has apparently been overlooked, and which bears on concepts arising from thin-airfoil theory. If the rotation $\alpha^* \equiv \alpha + \beta$, $\varphi^* \equiv \varphi - \beta$ is introduced, then by expanding and collecting terms,

$$U = k[\sin \varphi^* \cos \alpha^* + (1 + \cos \varphi^*)\sin \alpha^*] \tag{2}$$

so that at zero lift ($\alpha^* = 0$),

$$U_o = k \sin \varphi^* \tag{3}$$

and at $\alpha^* = \pi/2$,

$$U_1 = k(1 + \cos \varphi^*) \tag{4}$$

(The velocity distribution U_1 corresponds to the maximum dimensionless

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circulation to which the Kutta condition can give rise.) Evidently, then,

$$U = U_0 \cos \alpha^* + U_1 \sin \alpha^* \tag{5a}$$

and since lift coefficient is exactly proportional to $\sin \alpha^*$,

$$U = U_0 \cos \alpha^* + U_1 c_l / c_{l_1} \tag{5b}$$

where the final subscript 1 again refers to $\alpha^* = \pi/2$.

It is clear that, for small α^*, the two terms of either Eq. (5a) or (5b) are somewhat analogous to the thin-airfoil concepts of "basic" and "additional" velocity distributions, respectively. In fact, for symmetrical airfoils, the factor U_1/c_{l_1} is the same as the familiar "additional velocity increment" $\Delta v_a/V$ of Ref. 3. The correctness of Eq. (5b) for such airfoils has already been demonstrated by Loftin,[4] using the notation of Ref. 3, although $\Delta v_a/V$ was not interpreted as U_1/c_{l_1}. For cambered airfoils, of course, these two quantities are not the same, since $\Delta v_a/V$ is computed for uncambered thickness forms only, in accord with thin-airfoil concepts, and can no longer be used in an exact manner for cambered airfoils. Eqs. (5), however, are valid for all airfoils.

In thin-airfoil theory, the basic velocity distribution (that to which the "additional" is added) is chosen in an essentially arbitrary manner, usually at the "ideal" angle of attack[5] rather than at zero lift.[6] A somewhat similar arbitrariness can be demonstrated in exact theory by generalizing Eq. (5a). While the generalized equation is easily derived from Eq. (1), it may be instructive to use a different approach.

Let the "chord" of an arbitrary airfoil be a completely arbitrary reference line; that is, the flow at $\alpha = 0$ is any arbitrarily chosen flow which satisfies the Kutta condition, with surface-velocity distribution U_c. At $\alpha = \pi/2$, the "chord" is normal to free stream, and the velocity distribution may be designated U_n. At arbitrary α, then, the free-stream velocity vector may be resolved into components paralleled and normal to the "chord," and the distributions due to each component may be added. The result is

$$U = U_c \cos\alpha + U_n \sin\alpha \tag{6}$$

For small α, the concepts of "basic" and "additional" velocity distributions again have an approximate validity for the two terms of this equation. Only if the "basic" c_l is small, however, will the second term be approximately proportional to "additional" c_l. Furthermore, the concept that the effect of airfoil shape (either thickness, camber, or both) is primarily embodied in the "basic" velocity distribution finds no interpretation in the equations above.

Certain further concepts are suggested by the equations in their own right, and these may be of interest. The ratio c_l/c_{l_1} in Eq. (5b) is the ratio of any given lift coefficient to the maximum to which the Kutta condition can give rise. The term "circulation ratio" and symbol κ seem appropriate for this quantity. In terms of κ, Eq. (5b) becomes

$$U = \pm U_0 \sqrt{1 - \kappa^2} + U_1 \kappa \tag{5c}$$

The concept of circulation ratio might be useful in applications of boundary-layer control and forced circulation, in which ratios approaching unity, and effective ratios exceeding unity, respectively, might be encountered. The use of κ would lend emphasis to the theoretical nonlinearity and low slopes of lift curves for such flows, since $\kappa = \sin \alpha^*$ (when the Kutta condition is satisfied).

The use of the coordinate φ^* in Eqs. (2) to (4) serves to emphasize the unique conformal correspondence among all airfoils which maps trailing edges into trailing edges, and which includes the circle considered as an airfoil with "trailing edge" at $\varphi^* = \pi$. A characteristic of this conformal correspondence may be illustrated by considering the division of Eq. (4) by Eq. (3):

$$U_1/U_0 = \cot(\varphi^*/2) \tag{7}$$

This ratio is the same for all airfoils; furthermore, the ratio between the velocity distributions for any two absolute angles of attack α^* is likewise independent of airfoil geometry, as can be seen from Eq. (2). This means that, in principle, the conformal correspondence between any airfoil and a circle, and therefore between any two airfoils (e.g., the same airfoil with different flap deflections) can be established approximately from measured pressure distributions for two different angles of attack. From the same data, computations of pressure distribution for other angles of attack could also be carried out.

Finally, it may be of interest to consider the use of κ and α^*, U_0 and U_1 in computing flows in which both incidence and lift are specified (e.g., to agree with experiment), the Kutta condition and results near the trailing edge being ignored. This is done by replacing $\sin(\alpha+\beta)$ in Eq. (1) by the required circulation, made dimensionless with respect to the circulation for coincident stagnation points on the circle[1] - this is simply κ:

$$U = k[\sin(\varphi+\alpha) + \kappa] \tag{8}$$

From this, it is readily shown that

$$U = (U_0\cos\alpha^* + U_1\sin\alpha^*) - k(\sin\alpha^* - \kappa) \tag{9}$$

which gives the result as the sum of a Kutta condition term and a correction term.

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